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ON MATHEMATICAL PHYSICS BOUNDARY VALUE PROBLEM CORRECTNESS

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ABSTRACT

The problem analytical solution general method is considered on non-stationary filtration equations basis. The problem solving method is based on internal features regularization and reducing it to a problem with boundary conditions.

Keywords: gallery, local and total debits, internal and boundary conditions, sum of discontinuities of derived functions, one-way integral Laplace transform, methods of the theory of functions of a complex variable.

PROBLEM FORMULATION

In this paper, we state the boundary-value problem for parabolic equations system, interconnected by internal boundary conditions representing the functions derivatives discontinuities sum at some definition points. For it, statements are proved that solution existence evidence that continuously depends on boundary value problem parameters and conditions, and this confirms the above assumption. The proof is carried out by reducing the boundary value problem to Voltaire integral equations system type with respect to functions derivatives jumps that are a solution to differential equations system. The one-sided Laplace transform is applied to integral equations system. The transformed linear functional system solution is obtained by complex variable functions theory method.

The boundary and internal differential equations system, as well as the initial conditions, has the form [1,2]:

$$\frac{1}{\xi^{\alpha}} * \frac{\partial}{\partial \xi}\left(\xi^{\alpha} * \frac{\partial P_i}{\partial \xi}\right) = \frac{1}{ӕ_i} * \frac{\partial P_i}{\partial t}, 0 < \xi < 1, i = \overline{1,n};\ \mathrm{t} < 0\ (\alpha = 0 \text{ или } \alpha = 1); \quad (1)$$

$$P_i(\xi, 0) = \varphi_i(\xi) 0 < \xi < 1, \qquad i = \overline{1,n} \qquad (2)$$

$$\left.\frac{\partial P_i}{\partial \xi}\right|_{\xi=0} = \left.\frac{\partial P_i}{\partial \xi}\right|_{\xi=1} = 0, \qquad t > 0,\ \ i = \overline{1,n} \qquad (3)$$

$$\sum_{i=1}^{n} (2\pi\xi_j)^{\alpha} * K_i \left(\left.\frac{\partial P_i}{\partial \xi}\right|_{\xi=\xi_j+0} - \left.\frac{\partial P_i}{\partial \xi}\right|_{\xi=\xi_j-0}\right) = \sum_{i=1}^{n} q_{ij}(t) = q_j(t), j = \overline{1,n} \qquad (4)$$

$$P_i \in C^2\big((\xi_k, \xi_{k+1}) \times (o, \infty)\big) \ \cap C\big((0,1) \times (0, \infty)\big),$$

$$\varphi_i \in C^1(\xi_k, \xi_{k+1}) \cap C(0,1), \qquad k = 0,1, \dots, m;\ \xi_0 = 0,$$

$$\xi_{m+1} = 1, \qquad q_{i,j}(t) \in C^{'}(0, \infty), \qquad i = \overline{1,n};\ \ j = \overline{1,m}$$

Consider the following functional of the solution $\{P_i(\xi,t)\}_{i=1}^{n}$ of system (1) - (4) and an arbitrary time instant T>0.

$$J(T,\{P_i\}_{i=1}^{n}) = T + \max_{i,j} \sup_{t>T} \left|P_i(\xi_j,t) - P_{i+1}(\xi_j,t)\right|, \quad i=\overline{1,n}; \quad j=\overline{1,m} \qquad (5)$$

Theorem. For any $\varepsilon > 0$ there exists T=T(ε)>0 and such a solution $\{P_i\}_{i=1}^{n} = \{P_i(\varepsilon)\}_{i=1}^{n}$ of system (1)-(4), satisfying the conditions listed above that $J(T(\varepsilon),\{P_i(\varepsilon)\}_{i=1}^{n}) < \varepsilon$. Moreover, this solution continuously depends on the problem initial data from the problem initial data. $æ_i, K_i, \varphi_i(x), i=\overline{1,n}\,;\ \xi_j, q_i(t), j=\overline{1,m}$

It follows from this theorem that $inf_{\{P_{i=1}^{n}\}} J(T,\{P_i\}_{i=1}^{n}) = 0.$

Note that in the general case the solution $\{P_i\}_{i=1}^{n}$ of system (1)-(4), may not exist, which vanishes the functional (5) (naturally, it is equal to zero in this case). To do this, it would be necessary to impose too large restrictions on $\varphi_i(x), i=\overline{1,n}$, which do not correspond to real conditions.

Let $\alpha = 0$, we obtain the following boundary value problem:

$$\frac{1}{æ_i} * \frac{\partial P_i}{\partial t} = \frac{\partial^2 P_i}{\partial x^2}, 0 < \mathrm{x} < 1, \mathrm{t} > 0, i=\overline{1,n} \qquad (1`)$$

$$P_i(x,0) = \varphi_i(x), 0 < \mathrm{x} < 1, i=\overline{1,n} \qquad (2`)$$

$$\left.\frac{\partial P_i}{\partial x}\right|_{x=0} = \left.\frac{\partial P_i}{\partial x}\right|_{x=1} = 0, \qquad i=\overline{1,n} \qquad (3`)$$

$$\sum_{i=1}^{n} K_i \left(\left.\frac{\partial P_i}{\partial x}\right|_{x=\xi_j+0} - \left.\frac{\partial P_i}{\partial x}\right|_{x=\xi_j-0} \right) = \sum_{i=1}^{n} q_{ij}(t) = q_j(t), \quad j=\overline{1,m} (4`)$$

where P_i, φ_i, q_{ij} satisfy the piecewise smoothness properties formulated above.

Assuming that

$$U_i(x,t) = P_i(x,t) + \frac{x^2}{2K_i}\sum_{j=1}^{m} q_{ij}(t) - \frac{1}{K_i}\sum_{j=1}^{m} q_{ij}(t)(x-\xi_j)\theta(x-\xi_j)$$

where $\theta(x) = \begin{cases} 0, & x \le 0 \\ 1, & x > 0 \end{cases}$

Then $U_i(x,t)$ have continuous derivatives with respect to x and with respect to tв $(0,1) \times (0,\infty)$, $i=\overline{1,n}$. We get:

$$\frac{1}{æ_i} * \frac{\partial U_i}{\partial t} = \frac{\partial^2 U}{\partial x^2} + F_i, \qquad 0 < x < 1, \ t > 0, \ i=\overline{1,n} (6)$$

$$U_i(x,0) = U_0^i(x), \quad 0 < x < 1, \qquad i = \overline{1,n} \tag{7}$$

$$\left.\frac{\partial U_i}{\partial x}\right|_{x=0} = \left.\frac{\partial U_i}{\partial x}\right|_{x=1} = 0, \qquad i = \overline{1,n} \tag{8}$$

Where

$$U_i^0(x) = \varphi_i(x) + \frac{x^2}{2K_i}\sum_{j=1}^{m} q_{ij}(0) - \frac{1}{K_i}\sum_{j=1}^{m} q_{ij}(0)(x-\xi_j)\theta(x-\xi_j),$$

$$F_i(x,t) = -\frac{æ_i}{K_i}\sum_{j=1}^{m} q_{ij}(t) + \frac{x^2}{2K_i}\sum_{j=1}^{m} q_{ij}'(t) - \frac{1}{K_i}\sum_{j=1}^{m} q_{ij}'(t)(x-\xi_j)\theta(x-\xi_j)$$

The solution (6) - (8) will be:

$$U_i(x,t) = \int_0^1 \varphi_i(x)dx -$$

$$-\frac{æ_i}{K_i}\int_0^1 \sum_{j=1}^{m} q_{ij}(\tau)d\tau - \frac{1}{K_i}\sum_{j=1}^{m} q_{ij}(t)\left(\frac{1}{3} - -\xi_j + \frac{1}{2}\xi_j^2\right)$$

$$+ 2\sum_{i=1}^{\infty}\left(e^{-(\pi n)^2 x_i t}\int_0^1 \varphi_i(x)\cos\pi lx dx\right.$$

$$\left.+\frac{1}{(\pi i)^2 K_i}\sum_{j=1}^{m} q_{ij}(t)\cos\pi l\xi_j - \int_0^1\left(\sum_{j=1}^{m} q_{ij}(\tau)\cos\pi l\xi_j\right)e^{-(\pi l)^2 æ_i(t-\tau)}d\tau\right)\cos\pi lx$$

Then the solution (1`)-(3`) is:

$$P_i(x,t) = U_i(x,t) - \frac{x^2}{2K_i}\sum_{j=1}^{m} q_{ij}(t) + \frac{1}{K_i}\sum_{j=1}^{m} q_{ij}(t)(x-\xi_j)\theta(x-\xi_j), \; j = \overline{1,m} \tag{9}$$

From the form of this solution itself, its continuous dependence on the initial data obviously follows. Now we show that for any $\varepsilon > 0$ 0 it is possible to find such T$(\varepsilon) > 0$ and determine the local flow rates $q_{ij}(t)$, so that they continuously depend on the initial data of problem (1`)-(4`), satisfy condition (4`) and so that for functional (5) $J(T,\{P_i\}_{i=1}^n) < \varepsilon$ is satisfied, where $\{P_i\}_{i=1}^n$ is the solution corresponding to these rates. We impose on the solutions $P_i, i = \overline{1,n}$ the following conditions

$$P_i(\xi_j,t) - P_{i+1}(\xi_j,t) = r_{ij}e^{-S_{ij}t}, \qquad i = \overline{1,n}; \; j = \overline{1,m} \tag{10}$$

From condition (10) and formula (9) we obtain a system of Volterra equations of the first kind. The one-sided Laplace integral transform is applied to the system of integral equations [3]. The one-sided Laplace transform is one-to-one for the function zero for x<0, therefore, it can be argued that the solution of the system of Voltaire integral equations of the first kind exists and is unique ($\alpha = 0$). In conclusion, we note that there exists a solution $\{P_i(x,t)\}_{i=1}^{n}$ of system (1)-(4), for which $(0, \{P_i\}_{\tau=1}^{n}) = 0$ is necessary and sufficient for

$$\varphi_i(x_i) = \varphi_{i+1}(x), i = \overline{1, n-1}, j = \overline{1, m}.$$

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