# FORMATION OF ALGORITHMS FOR ESTIMATING UNKNOWN INPUT SIGNALS IN DYNAMIC CONTROL SYSTEMS

**[1]Kodirov Dilmurod Tukhtasinovich, [2]Kodirova Feruza Mukhammadzhanovna, [3]Sharibboev Shokhjakhon**

Associate Professor of the Department of Automation and Control of Technological Processes and Production and Information Technologies of the Namangan Institute of Engineering and Technology[1], Senior lecturer of the department "Building materials and products" of the Namangan Civil Engineering Institute[2], Master student of the Department of Automation and Control of Technological Processes and Production and Information Technologies of the Namangan Institute of Engineering and Technology[3]

---

## ABSTRACT

The paper considers the formation of regularizing algorithms for recovering unknown input signals in control systems based on approximate information of the measure of incompatibility of the original operator equations using an estimate that cannot be improved on the class of input data equivalent in accuracy.

***Key words: regularization, accuracy, stability, linear equations.***

The problem of restoring the initial state and input action of a dynamic system from the results of measuring the output belongs to the class of inverse problems of the dynamics of controlled systems. Since the indicated problem is ill-posed, for its solution one should apply the methods developed in the corresponding theory [1].

Let us consider a linear dynamical system with observation:

$$x_{k+1} = A_k x_k + B_k w_k, \ x(k_0) = x^0, \qquad (1)$$

$$y_k = C_k x_k + D_k w_k, \qquad (2)$$

Where $x \in R^n, w \in R^p, y \in R^m$; $x = x_k$ is state of the system; $x^0$ *is initial state of the system;* $w_k \in L_2^p$ is input unmeasured impact on the system; $y_k \in L_2^m$ is system output; and $A_k, B_k, C_k, D_k$ is matrices of the corresponding dimensions.

Let it be $\Theta = R^n \times L_2^p, \ Y = L_2^m$. Having defined $\Theta$ is the scalar product in space.

$$\langle \theta_1, \theta_2 \rangle_\Theta = \langle x_1^0, x_2^0 \rangle_{R^n} + \langle w_1, w_2 \rangle_{L_2^p},$$

let us turn the space $\Theta$ into a Hilbert space.

Relations (1), (2) define a linear operator is $F : \Theta \to Y$, which for each pairis $\theta = (x_0, w) \in \Theta$, i.e. system input, assigns a function $y \in Y$ at the system output. Let be $y^*$ is some output of system (1), (2). We denote $\Theta^*$ by the non-empty set of all inputs is $\theta \in \Theta$ such that:

$$F\theta = y^*. \qquad (3)$$

It is necessary $y^*$ to restore $\Omega$ is the output to a normal input compatible with this output. Suppose that we do not know the $y^*$ output, but we know the result of measuring the output, is $y_\delta \in Y$ such that:

$$\| y_\delta - y^* \|_Y \le \delta,$$

Where: $\delta$ is a well-known non-negative parameter characterizing the accuracy of the measurements. In this case, it is impossible to accurately reconstruct the $\Theta^*$ set, and even more $\theta^* \in \Theta^*$.so the element Taking this circumstance into account, let us consider the problem of approximate recovery of the element $\theta^* = (x_*^0, w^*)$ from inaccurate yield measurements of $y^*$ assuming, that $A, B, C, D$ the matrices and the functional $\Omega$ are known exactly. By function of $y_\delta$ and parameter of $\delta > 0$ need to find a such pair for $\theta_\delta = (x_\delta^0, w_\delta(\cdot)) \in \Theta$ so that $\|\theta_\delta - \theta^*\|_\Theta \to 0$ is $\delta \to 0$.

In the general case, system (3) is inconsistent and ill-conditioned. Therefore, for a stable construction of a pseudo solution, we will use the regularization method.

Let a pair of numbers be given $\eta = (h, \delta)$, $h \ge 0$, $\delta \ge 0$, and an approximate input data $p_\eta = (F_h, y_\delta^*) \in W$, satisfying the inequalities.

$$\|F_h - F_0\| \le h, \quad \|y_\delta^* - y_0^*\| \le \delta,$$

Consider a class of input data equivalent in accuracy [2]:

$$\Sigma_\eta = \{ p = (F, y^*) \in W : \|F - F_h\| \le h, \ \|y^* - y_\delta^*\| \le \delta.$$

It follows from definition (4) that an estimate $\mu_D[F_0, y_0^*]$ that cannot be improved in the class of input data equivalent in accuracy is the value

$$p_\eta(F_h, y_\delta^*) = \sup_{p \in \Sigma_\eta} \mu_D[F, y^*] = \sup_{p \in \Sigma_\eta} \inf_{\theta \in D} \|F\theta - y^*\|. \quad (5)$$

Moreover $p_\eta(F_h, y_\delta^*) \ge \mu_D[F_0, y_0^*]$, is the equality $\lim_{\eta \to 0} \rho_\eta[F_h, y_\delta^*] = \mu_0[F_0, y_0^*]$.

However, the calculation $p_\eta(F_h, y_\delta^*)$ in the general case is difficult. In [1], it is proposed to consider the problem dual to (5)

$$\tilde{p}_\eta(F_h, y_\delta^*) = \inf_{\theta \in D} \sup_{\chi \in \Sigma_\eta} \|F\theta - y^*\|.$$

The results imply the inequality

$$p_\eta(F_h, y_\delta^*) \le \tilde{p}_\eta(F_h, y_\delta^*),$$

$$\Phi_\eta[z] = \sup_{p \in \Sigma_\eta} \|F\theta - y^*\|, \qquad \theta \in D.$$

For any $F_h \in L$, $y_\delta^* \in U$, $\theta \in D$, $h \ge 0$, $\delta \ge 0$ there are $(\bar{F}, \bar{y}^*) \in \Sigma_\eta$ such [3] that

$$\|\bar{F} - F_h\| = h, \quad \|\bar{y}^* - y_\delta^*\| = \delta,$$

$$\Phi_\eta[\theta] = \|\bar{F}\theta - \bar{y}^*\| = \|F_h\theta - y_\delta^*\| + h\|\theta\| + \delta$$

Thus

$$\tilde{p}_\eta(F_h, y_\delta^*) = \inf_{\theta \in D} (\|F_h\theta - y_\delta^*\| + h\|\theta\| + \delta) = \inf_{\theta \in D} \Phi_\eta[\theta]. \quad (6)$$

Consider an algorithm $\Phi_\eta[\theta]$ for minimizing the functional on the entire space Z. According to [3], for D = Z problem (6) is equivalent to the problem of minimizing the Tikhonov's functional:

$$M_\alpha[\theta] = \left\|F_h\theta - y_0^*\right\|^q + \alpha\|\theta\|^r, \quad q \geq 1, \quad r > 1,$$

with the choice of the regularization parameter from the "principle of the least residual estimate":

$$\psi(\alpha) = \left\|A_h\theta_\alpha - y_0^*\right\| + h\|\theta_\alpha\| \to \min, \quad \theta_\alpha = \arg\min_{\theta \in Z} M_\alpha[\theta]. \tag{7}$$

The function $\psi(\alpha)$ reaches its minimum at $[0,+\infty)$ at a single point $\alpha_0$, and $\theta_{\alpha 0} = \theta_\eta$.

The method of the least estimate of the residual is equivalent to the following principle of the least estimate of the residual [4]. If

$$h\left\|y_\delta^*\right\| \geq \left\|F_h^T y_\delta^*\right\|, \tag{8}$$

then we assume $\theta_\eta = 0$; otherwise, we solve the equation;

$$\left(F_h^T F_h + \alpha I\right)\theta = F_h^T y_\delta^* \tag{9}$$

with the choice of the regularization parameter is $\alpha \geq 0$ from condition (7); moreover, the function is $\psi(\alpha)$ continuously differentiable for $\alpha > 0$, has a unique point $\alpha_0$ of local minimum, which is also a point of global minimum on $[0,+\infty)$, and a vector is $\theta_{\alpha_0} = \theta_\eta$. If then the only solution to the equation is:

$$\alpha\|\theta_\alpha\| = h\left\|F_h\theta_\alpha - y_\delta^*\right\|;$$

if $\alpha_0 = 0$, then for any $\alpha > 0$ the following inequality is valid:

$$\alpha\|\theta_\alpha\| - h\left\|F_h\theta_\alpha - y_\delta^*\right\| > 0.$$

At this, if $F_h\theta_\eta \neq y_\delta^*$ and $\theta_\eta \neq 0$, then the vector $\theta_\eta$ is a solution to the equation

$$\frac{F_h^T F_h\theta - F_h^T y_\delta^*}{\left\|F_h\theta - y_\delta^*\right\|} + h\frac{\theta}{\|\theta\|} = 0$$

and any solution to this equation minimizes the $\Phi_\eta[\theta]$ functional.

The equivalence of the method of the least estimate of the residual to problem (8), (9) and the uniqueness of the minimum point $\alpha_0$ follows from (7). The function $\psi(\alpha)$ is continuously differentiable for $\alpha > 0$ and

$$\psi^T(\alpha) = 0.5\left(\left(F_h^T F_h + \alpha I\right)^{-1}\theta_\alpha, \theta_\alpha\right)\left(\frac{\alpha}{\left\|F_h\theta_\alpha - y_\delta^*\right\|} - \frac{h}{\|\theta_\alpha\|}\right),$$

where the operator $\left(F_h^T F_h + \alpha I\right)^{-1}$ is positively definite.

Using transformations similar to the transformations from [5], the regularized normal system of equations (9) can also be represented in the form of an equivalent extended regularized normal system of equations:

$$\begin{bmatrix} \omega I_m & F \\ F^T & -\omega I_n \end{bmatrix}\begin{bmatrix} u \\ \theta \end{bmatrix} = \begin{bmatrix} y^* \\ 0 \end{bmatrix} \Leftrightarrow \tilde{F}_\omega x = \tilde{y}_\omega^*, \tag{10}$$

where $u = \omega^{-1} r$, $r = y^* - F\theta$, $\omega = \alpha^{1/2}$.

The eigenvalues of the matrix $\tilde{F}_\omega$ are numbers $\pm\sqrt{\sigma_i^2 + \omega^2}$, $i = 1,\ 2,...,\tau$, as well as $\omega$ and $-\omega$ multiplicities of $m - \tau$ and $n - \tau$, respectively, where $\tau = rank(F) \geq 1$ and $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_\tau$ are the singular values of the matrix $F$.

The solution of $x_\omega = \begin{bmatrix} u_\omega \\ \hline \theta_\omega \end{bmatrix}$ to the extended regularized system (10) exists, is unique and is equal to:

$$\theta_\omega = (F^T F + \omega^2 I_n)^{-1} F^T y^*,$$

and $u_\omega = \omega^{-1} r_\omega$, where $r_\omega = y^* - F\theta_\omega$.

Regardless of the rank of the matrix, the upper bound for the spectral condition number of the extended regularized system (10) holds:

$$\kappa_2(\tilde{F}_\omega) \leq \frac{\sqrt{\sigma_1^2 + \omega^2}}{\omega}.$$

This fact means that the condition number of the extended regularized system (10) is much less than the condition number of the normal system (9) [5]. Consequently, for values $\omega << 1$, the solution of $\theta_\omega$, obtained from the solution to the extended system (10) will be much more accurate than the corresponding solution based on the ordinary normal system (9).

If the original system of linear equations $F\theta = y^*$ is inconsistent, then it is possible that $\|\tilde{u}\|_\infty >> \|\tilde{\theta}\|_\infty$. In this case, the accuracy of calculating the vector $\tilde{u}$ can be much higher than the accuracy of the vector $\tilde{\theta}$ and will be close to the accuracy of the entire solution vector $\tilde{x}$ of the extended system (10).

In this case, it is advisable to correct the solution of the $\tilde{x}$ extended normal system of linear equations (10). For this, the estimate of the projection of the vector $\tilde{y}^*$ onto the image F is calculated:

$$\tilde{y}^* = y^* - \omega\tilde{u}$$

The above expressions make it possible to form regularizing algorithms for recovering unknown input signals in control systems from approximate information of the measure of incompatibility of the original operator equations using an estimate that cannot be improved on a class of input data equivalent in accuracy.

## REFERENCES

[1] Shankar P. Bhattacharyya, Lee H Keel. Control of Uncertain Dynamic Systems. CRC Press. 1991. -544 p.

[2] Karabutov N.N. Structural identification of systems. Analysis of information structures. Publishing house: Librokom, 2009 .-- 176 p.

[3] Krutko P.D. Inverse problems of dynamics in the theory of automatic control. –M .: Mechanical engineering, 2004. - 576 p.

[4] Golubev A.E., Krishchenko A.P., Tkachev S.B. Stabilization of Nonlinear Dynamical Systems Using the Estimation of the System State by an Asymptotic Observer (Review) // Autom. 2005, no. 7. –S.3–42.

[5] Krasnova S.A. Cascade synthesis of a state observer for nonlinear systems in the presence of external disturbances. Avtom., 2003, no. -WITH. 3-26